Who was Euclid ?

Marina Brown

Who Was Euclid ?

Euclid was a professor who taught mathematics, science and geometry in the Greek city of Alexandria in Egypt. Most importantly he was a product of his times and of the great city in which he lived. Alexandria was built between 332 and 321 BCE by the Greek architect Dinocrates. The city like Athens was a center of learning the drew the brightest minds of the time to study and write. The city was the home of the tallest building of the time, which was the lighthouse. The lighthouse had a mirror that cast light 35 miles onto the Mediterranean and guided sailors into the harbor. (12) Alexandria was home to the Greatest Library of the Ancient world, one of the greatest Museums, and temples built by the finest architects. Alexandria thrived as a center of learning from the time it was built until the 5th century when philosophy and learning were seen as a threat to the newly emerging religion of Christianity. Alexandria weathered the conflict between Julius Caesar and Pompey. There are many conflicting accounts of the burning of the library but it was probably during this conflict that the Great Library was destroyed. The Library allegedly contained Aristotle's private library and many other great texts that are lost to us today. Historians and philosophers have written that the library contained as many as 700,000 texts. Alexandria continued as a center of ancient learning until around the 4th century CE. It's gradual decline began when it was destroyed by Diocletian in retaliation for an imagined insult. Intolerance of pagan learning by the new Christian rulers of the Mediterranean led to the burning of the Museaum and the Serapaum by Theodosius the Patriarch of Alexandria. One of the saddest incidents in history signaled the death of learning in the West was the murder of Hypatia, the Daughter of the last keeper of the museum at Alexandria. She was one of the last pagan mathematicians and scientists in the west.

During Alexandria's heyday Eretosthenese had calculated the diameter of the earth to within 1% by measuring the difference in the angle of the noonday sun in distant cities. It would take centuries and the persecution of Galileo before west would again understand that the earth was spherical. All in all Alexandria was a shining light of learning for almost 700 years. It was this city that Euclid called "home".

Euclid's lived from 325 BCE to around 265 BCE. His contemporaries included Eretosthenese, Eudemus of Rhodes, Autolycus of Pitane. He was too young to have studied with Plato, but many of Plato's students lived at the same time as Euclid. For the most part Euclid's though is Platonic. For a Platonist the reality we see around us is merely a shadow of the real truth which lies in the realm of pure thought. The Mac history archives at the School of Mathematics and statistics says this about Euclid:

> "In his aim he was a Platonist, being in sympathy with this philosophy, whence he made the end of the whole "Elements" the construction of the so-called Platonic figures". [1]

[1] http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Euclid.html

To the same ends Euclid and Plato were very close to being Pythagoreans. Pythagoras is known for his teaching the equation $a^2+b^2=c^2$ a and b being the legs of a right angle triangle c being the hypotenuse. This equation has applications in all sorts of physical phenomenon. Einstein's famous $E = MC^2$ equation can be derived from $a^2+b^2=c^2$.(8) Pythagoras is less well known for his belief that mathematics is the basis of reality.

He believed:

1. that at its deepest level, reality is mathematical in nature,
2. that philosophy can be used for spiritual purification,
3. that the soul can rise to union with the divine,
4. that certain symbols have a mystical significance, and
5. that all brothers of the order should observe strict loyalty and secrecy.

[2]

To a great extent Plato's belief that thought and logic were the true reality are contained in the principles of Pythagoras's school. Euclid followed in the footsteps of Plato and Pythagoras.

Like modern day professors, Euclid had many publications. His most famous and monumental publication was "The Elements". The Elements is composed of 13 books on the subjects of plane geometry, circles, number theory, irrational numbers and three dimensional geometry. His other publications include "Data", "On Divisions", "Optics", "Phaenomena", "Porisms", "Conics", "Porisms", "Surface Loci",and "Book of Fallacies and Elements of Music". Of these works 3 are lost to antiquity. What we know of them is from the writings of other mathematicians, philosophers and historians.

In "The Elements" Euclid laid out much of the mathematical wisdom of the ages before him. His work is a compilation of principles of the many mathematicians who preceded him. It is tempting to crown Euclid with the title "The Father of Geometry", but this would be a great injustice to the many mathematicians whose work he draws upon. Although travel was difficult in the ancient world people did travel extensively. Pythagoras traveled to Egypt and to Babylonia where he inevitably learned from philosophers and mathematicians. Much earlier proofs of "The Pythagorean Theorem" have been found on Babylonian clay tablets. What Euclid did was to perfect and compile earlier wisdom in a methodical fashion that inspired and challenged generations of students and scholars.

[2] http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html

In all of the books of “The Elements” Euclid started by putting forth a number of postulates, propositions and definitions that he uses in his proofs. In book 1 he creates 5 postulates, 23 definitions and 48 propositions that allow him to discuss plane geometry.

One of the most famous and controversial of Euclid’s definitions is definition 34 in book 1 that states that:

> “Parallel straight lines are straight lines which, being in the same plane and being produced indefinitely in both directions, do not meet one another in either direction. ”(4)

Book 1 also states in Common notion 1:

“Things which equal the same thing also equal one another.” (4)

This definition stood for about 2000 years unquestioned until Gerolamo Sacceri attempted to disprove Euclid by postulating that there were no parallel lines. Sacceri’s proofs were completed by Hilbert in the 20th Century(5). Sacceri postulated that since points were infinitesimal the only way to have 2 parallel lines was to draw the two lines between the same 2 points.

The most convincing challenge to Euclidean geometry came from Bernhard Riemann in the mid 19th Century. Riemann proposed a geometry that was not based on the Straight lines and “Flat” space upon which Euclid based all his work. In Riemannian Geometry space itself is curved. Euclidian principles work within an infinitesimal frame, but on a larger scale they don’t hold as space itself is curved. In Riemannian geometry like the geometry of the surface of a sphere a straight line is a geodesic. In a spherical or elliptical space all lines DO meet. One of the best description of curved space within a Riemannian framework was in the book “Gravitation” written by Wheeler, Misner and Thorne in 1979. In the parable of the ant, the reader is asked to imagine a tiny 2 dimensional ant on the surface of an apple. The ant being rather intelligent makes the Euclidean assumptions that a straight line will go on forever. Since the ant is 2 dimensional it has no direct understanding of the 3 dimensional nature of the surface of the apple. It has full freedom to move in straight lines in it’s 2 dimensional space. Like an imaginary space traveler all it sees is unending space. The ant sets out to investigate the nature of his space by walking over the apple in straight lines. The ant is walks a geodesic on the surface of the apple that is spherical and is surprised to find itself back at the spot where it started. When the ant tries walking parallel lines it is surprised to find that after a certain distance the straight lines intersect. The ant is forced to come to the conclusion that what he once thought was infinite space was finite, but unbounded. While Euclidean assumptions still hold with an infinitesimal degree of error in a small local frame, they don’t hold in a curved space on a large scale. In a non-infinitesimal scale Euclid's definition that parallel lines never meet does not hold in a positively curved space as described by Riemann. Because of the irregularities on the surface of the apple, the ant would be force to come to differing conclusions depending on the curvature of the apple. On a flat area, the and would come to the conclusion that the sum of the interiour angles of a triangle was 180 degrees. On a positivly curved area, the ant would find the sum to be more than 180 degrees. On a negativly curved area, it would find the sum to be less.

In 1879 Michelson first measured the speed of light. After establishing the speed of light he then set about to see if light was effected by motion through a medium that scientists of the time called aether. Light was known to be a wave so Michelson assumed that the earth moving on it's orbit would effect the speed of light. The amazing result of the experiment was that light moved at the same speed REGARDLESS of the frame in which it's speed was measured. (6) Scientist set about to analyze how this could happen. What they figured out through an application of Pythagoras's theorem was that object moving were foreshortened when seen from a reference that saw them in motion. This allows for light moving at the same speed within any frame of reference. Most scientist of the time were not ready to accept the implications of this conclusion. The first scientist who thoroughly examined this conclusion was Einstein. The theory he created was known as "Special Relativity". In Special Relativity, Euclidean principles hold within a local frame of reference and can be used. In Special Relativity the common notion that "Things which equal the same thing also equal one another." does not hold in differing frames of reference. Einstein added the qualification that things are equal within the same inertial frame.

In 1911 Einstein first measured the the bending of light by a gravitational field. To do this he waited for an eclipse and observed a star whose light passed close to the sun. There was a small but measurable deflection towards the sun. In special relativity the distance light travels in a given time became the universal measuring stick. The fact that light itself was effected by gravitation led Einstein to the conclusion that gravitation was not a force, but a manifestation of the curvature of space. Einstein postulated that matter curves space like a weight on a taught sheet of rubber. The deflection of light was like the two dimensional ant on an apple following the geodesics. The question that intrigues scientist to this day is the nature of this curvature. Is there enough matter to close space into a sphere that will eventually collapse upon itself or will space continue to expand indefinitely until everything is evenly distributed and there are no more possibilities. In addition to his studies on gravitation Einstein studied the new field of quantum mechanics. Einstien abandoned the study of quantum physics for a period. He is commonly quoted as saying "God does not play dice". Quantum Physics predicts that all particles including matter that is collapsed into a point by gravitation will decay into energy. Einstein's theory of General Relativity like Euclid's geometry was found to have limits in it applicability. Today we use Pythagoras's theorem, Euclid's geometry and General Relativity in every day applications. We might use Pythagoras's theorem to calculate the speed of an airplane through a crosswind, in laying kitchen tiles a basic understanding of Euclid's principles is a great help. Global Positioning Systems (GPS) are used by hikers by rescue workers, in cars, airplanes and boats for navigation. GPS technology depends on extremly precise clock synchronization. It is not possible to maintain the synchronization of clocks in differing inertial frames and gravity without an understanding of Special and General Relativity. Despite the limitations of these theories they find essential uses.

In 1931 a mathematician who like Einstein spent time at Princeton University published "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme". This work contained Godel's famous "Incompletness theories" . (9) In a mathematical sense the Incompletness Theories showed:

"He proved fundamental results about axiomatic systems showing in any axiomatic mathematical system there are propositions that cannot be proved or disproved within the axioms of the system. In particular the consistency of the axioms cannot be proved."

from: `http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Godel.html`

In common language Godel showed that no formal theory can explain all phenomenon. Theories have a limited range of validity that is limited by their own complexity. Godel's theorem in and of itself contains Euclids first common notion "Things which equal the same thing also equal one another." (`http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html`#posts). This is the principle of identity ie a = a. Applying Godel's incompleteness theorems to complex natural phemomena one comes to the conclusion that the only thing that can fully explain a phenomenon is the phenomena itself. We can create usefull theories that help us to understand limited portions of reality. We must be very careful in creating universal rules because all theories have limitations to their applicability. (10)(11)

Despite the fact that Euclidean geometry has proven not to apply to all circumstances, it's impact upon human thought cannot be overlooked. Almost all mathematicians and physicists build upon Euclid's work and compilation of works on some level. In most everyday applications Euclids geometrical works and the application of Euclidean geometry to physics, Newtonian Physics work well. In many non-euclidean systems Euclidian principles are used on the infinitesimal level to build their conclusions. If all of our notions and theories are merely shadows on the wall of our minds, the shadow that Euclid wrote about is one of the most remarkable shadows.

Footnotes

(1) `http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Euclid.html`
(2) `http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html`
(3) `http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html`#posts

(4) http://aleph0.clarku.edu/~djoyce/java/elements/bookI/defI23.html
(5) http://www.friesian.com/curved-1.htm
(6) Gravitation by Wheeler, Misner and Thorne - W.H.Freeman & Co., 1971
(7) http://galileoandeinstein.physics.virginia.edu/lectures/michelson.html
(8) From Pythagoras to Einstein - K. O. Friedrichs - 1965
(9) http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Godel.html
(10) http://www.clausewitz.com/CWZHOME/CWZBASE.htm
(11) Gödel, Escher, Bach - an Eternal Golden Braid by Douglas Hofstadter
(12) PBS special on the "7 wonders of the ancient world" around 2002

bibliography

http://grnwd.tripod.com/overthrow.html
http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Euclid.html
http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Pythagoras.html
http://aleph0.clarku.edu/~djoyce/java/elements/bookI/bookI.html#posts
http://aleph0.clarku.edu/~djoyce/java/elements/bookI/defI23.html
http://www.friesian.com/curved-1.htm
http://galileoandeinstein.physics.virginia.edu/lectures/michelson.html
http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Godel.html
http://www.surferz.net/~marina
Gravitation by Wheeler, Misner and Thorne - W.H.Freeman & Co., 1971
From Pythagoras to Einstein - K. O. Friedrichs - 1965
On War - Clausewitz

colophone

This paper was initially typeset for the web using the vi editor. It's HTML was converted to LaTeX with latex2html which can be found at http://html2latex.sourceforge.net. It was converted to a booklet using psutils found at http://gershwin.ens.fr/vdaniel/Doc-Locale/Outils-Gnu-Linux/PsUtils/